Rotating convection

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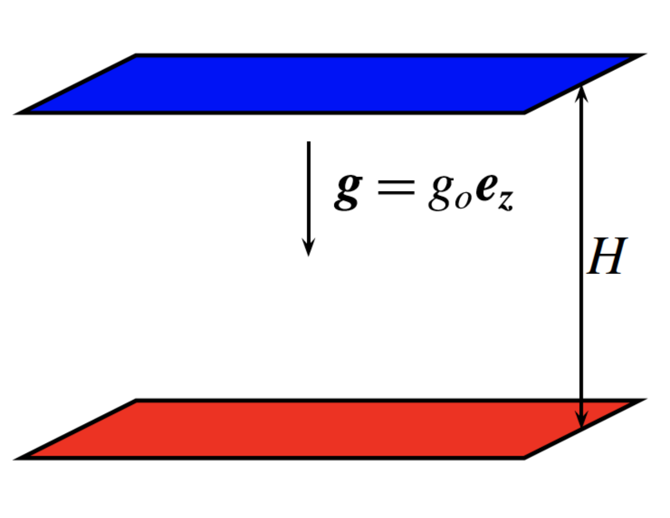
Section 1

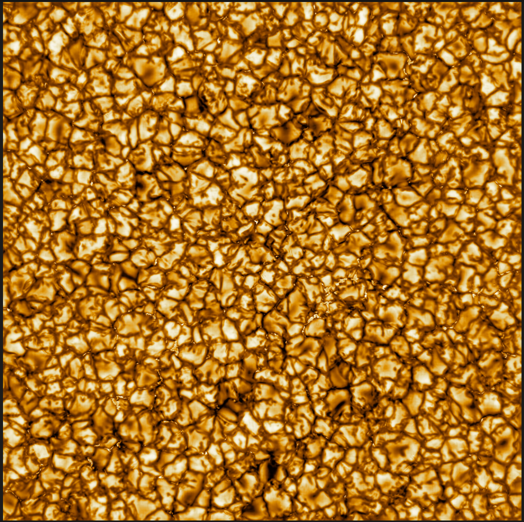
Introduction

Convection in a box



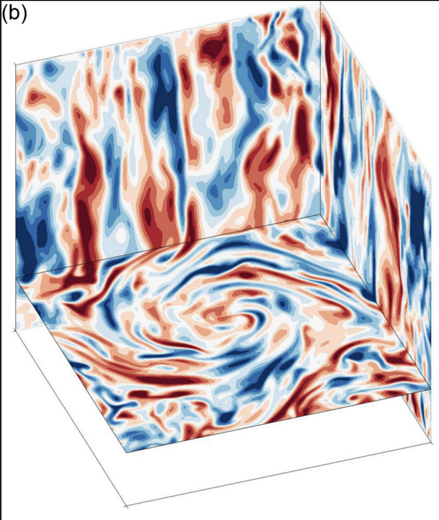
(Gastine et al., 2015)

The sun



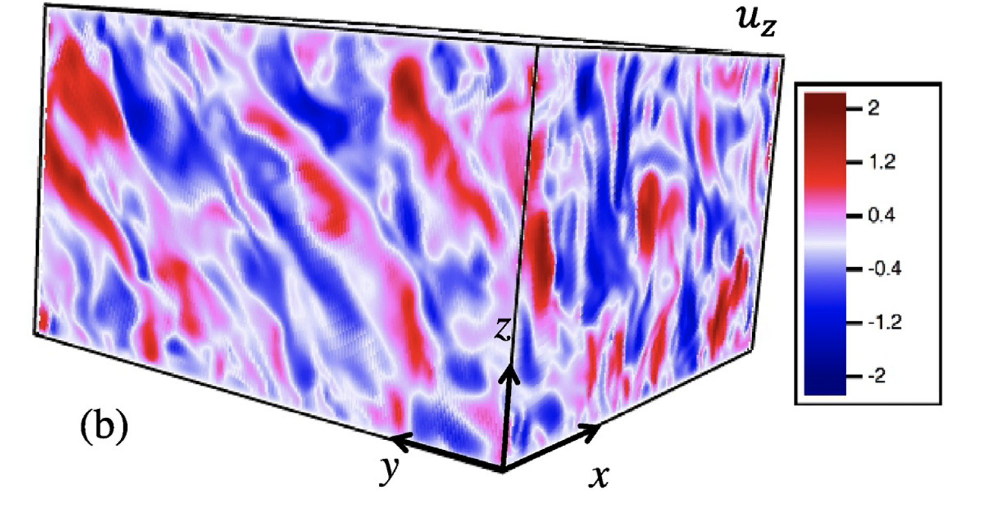
DKIST telescope, credit: NSO/NSF/AURA

Add rotation



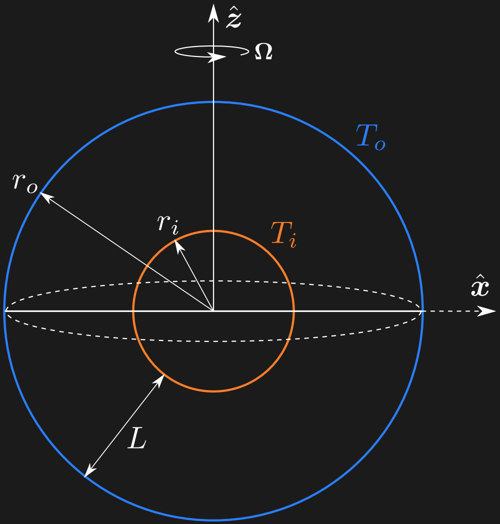
(Guervilly and Hughes, 2017)

Rotation + gravity misaligned

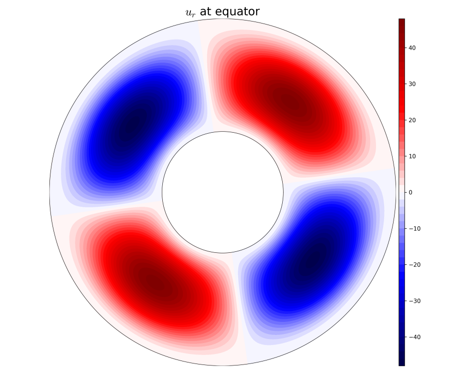


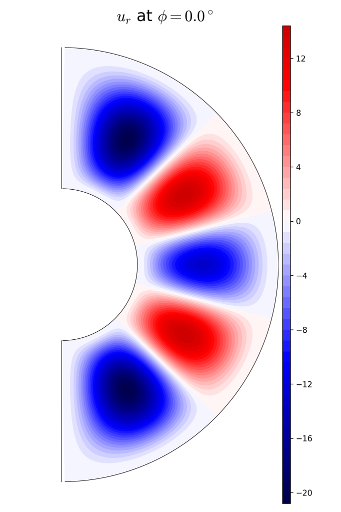
(Currie et al., 2020)

Onset of convection in a spherical shell

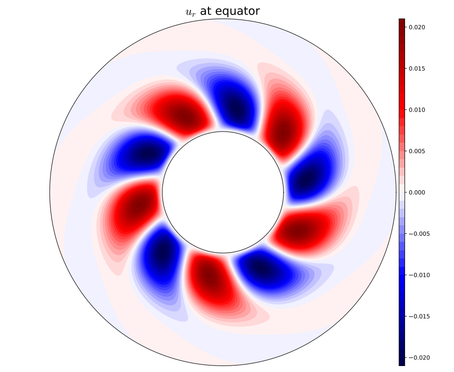


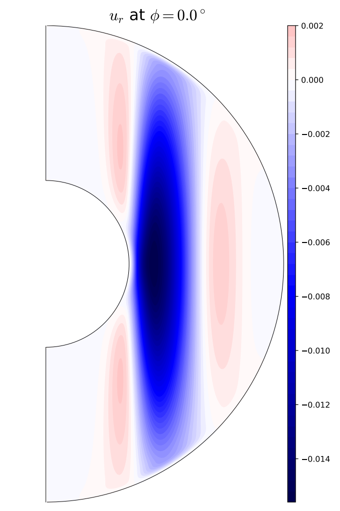
Non-rotating case





Rotating case





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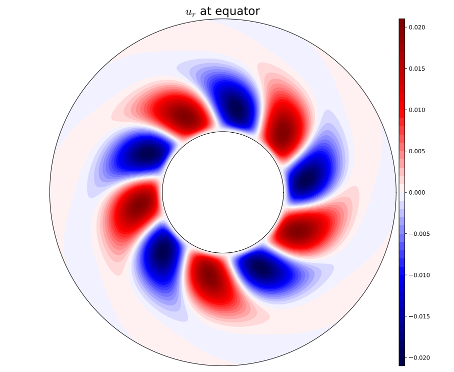
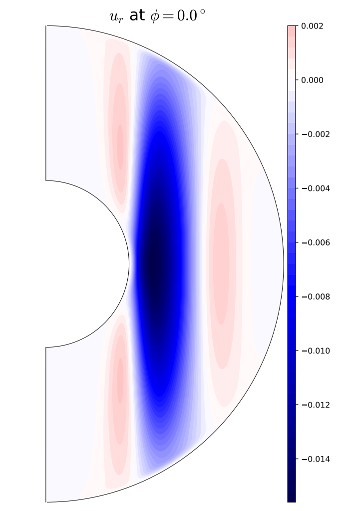
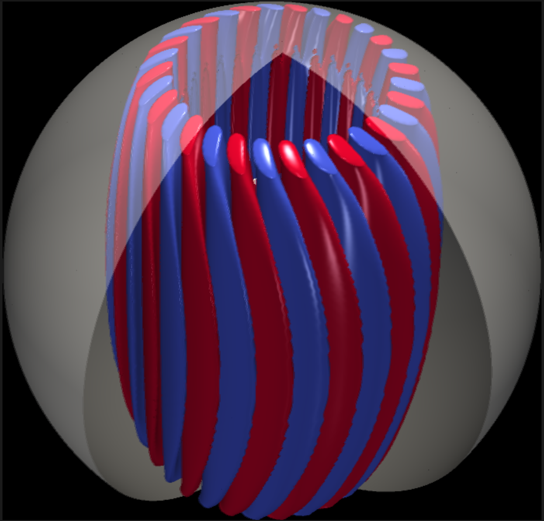




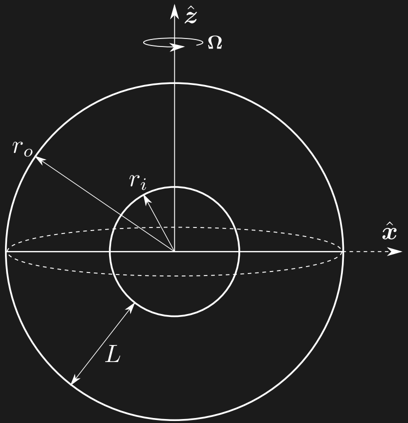
$$m = 5$$

Rotating case



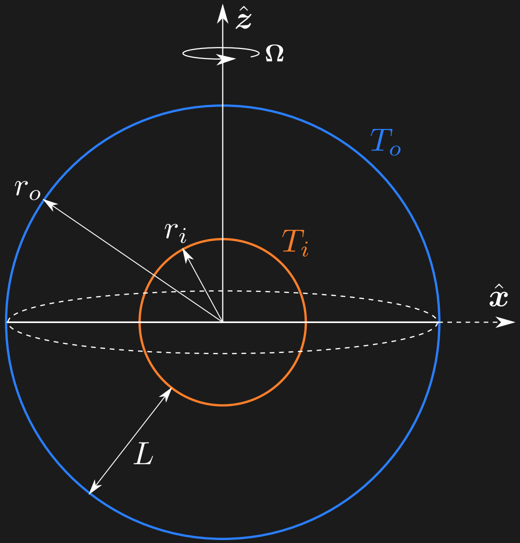
$m = 15$

Onset of convection in rotating spherical shells



- Classical fluid mechanics problem (Chandrasekhar, 1961)
- Subsequent theoretical studies by e.g: Roberts (1968); Busse (1970, 1986); Zhang and Busse (1987); Yano (1992); Dormy et al. (2004); Al-Shamali et al. (2004); Calkins et al. (2013)
- Experimental study by Carrigan and Busse (1983)

Equilibrium



Fixed temperature at boundaries:
$\Delta T = T_i - T_o$

$$\frac{dP}{dr} = -\rho g(r)\hat{\boldsymbol{r}},$$

$$\nabla^2 \langle T \rangle = 0$$

$$\Rightarrow \frac{d\langle T \rangle}{dr} = \frac{r_i r_o}{r_o - r_i} \Delta T \frac{1}{r^2}$$

Perturbations

$\boldsymbol{u}$ = velocity
T = temperature
p' = pressure

$$(\boldsymbol{u}, T, p') \equiv (\boldsymbol{u}, T, p')\, e^{im\phi + \lambda t}$$

m = azimuthal wavenumber/symmetry

$$\lambda = \sigma + i\omega$$

σ = growth/decay rate
ω = drift frequency

Equations to solve

Scales: Length : $L = r_o - r_i$, Time : L^2/ν, Temperature: $\Delta T = T_i - T_o$

$$\frac{\partial \boldsymbol{u}}{\partial t} = \lambda \boldsymbol{u} = -\nabla p' - \frac{2}{E}\hat{\boldsymbol{z}} \times \boldsymbol{u} + \frac{Ra}{Pr} r T \hat{\boldsymbol{r}} + \nabla^2 \boldsymbol{u}$$

$$\frac{\partial T}{\partial t} = \lambda T = -u_r \frac{dT}{dr} + \frac{1}{Pr}\nabla^2 T$$

$$\nabla \cdot \boldsymbol{u} = 0$$

Equations to solve

$$\lambda \begin{bmatrix} \boldsymbol{u} \\ T \end{bmatrix} = \begin{bmatrix} \cdots & & \cdots \\ & \ddots & \\ \cdots & & \cdots \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ T \end{bmatrix}$$

Non-dimensional control parameters

Parameter	Numerical studies	Astrophysical bodies
Radius ratio, $\chi = \frac{r_i}{r_o}$	0.05 to 0.95	Varied
Ekman number, $E = \frac{\nu}{\Omega_o L^2}$	$10^{-7\,*}$ to 10^{-3}	$\sim 10^{-10}$ to 10^{-15}
Prandtl number, $Pr = \frac{\nu}{\kappa}$	1	$10^{-2} - 1$
Rayleigh number, $Ra = \frac{\alpha g_o \Delta T L^3}{\nu\kappa}$	10^{10}	10^{25}

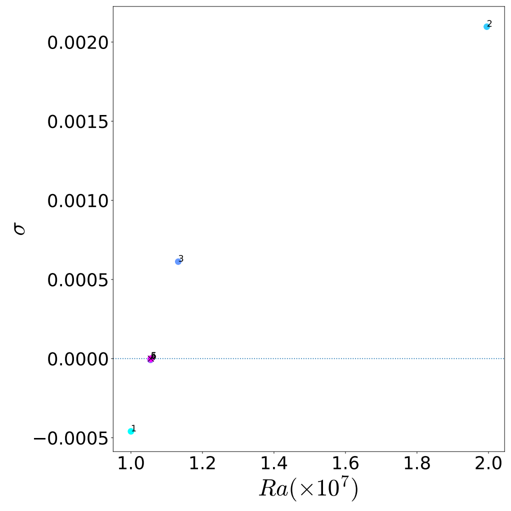
Spectral method

Resultant problem is a generalized eigenvalue problem:

$$\mathcal{A}\,x = \lambda\,\mathcal{B}\,x$$

where, $x \equiv \begin{bmatrix} \boldsymbol{u} \\ T \end{bmatrix}$ is a vector of coefficients and $\lambda = \sigma + i\omega$.

Search for critical parameters



$$(\boldsymbol{u}, T) \sim e^{im\phi + (\sigma + i\omega)t}$$

1. Fix E and $\chi = r_i/r_o$
2. Find critical Rayleigh numbers Ra for onset of convection $(\sigma = 0)$ for a fixed m
3. Repeat for a range of m
4. Pick the minimum in Ra to obtain Ra_c and m_c

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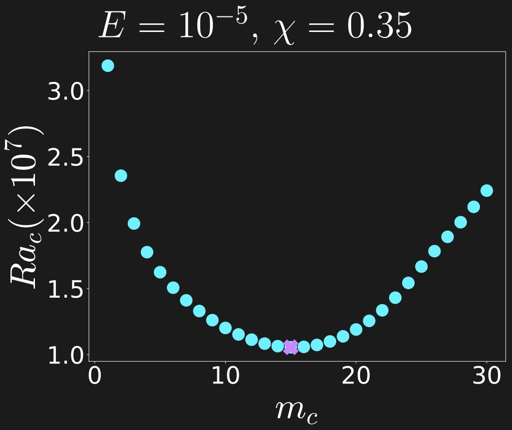

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Scalings from linear theory

$$m_c \sim E^{-1/3},\ Ra_c \sim E^{-4/3},\ \omega_c \sim E^{1/3}$$

What happens when Ra is increased?

Heat transfer scalings:
Non-rotating turbulent Rayleigh-Bénard

$$Nu \sim Ra^{1/3} \tag{1}$$

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Heat transfer scalings:
Non-rotating turbulent Rayleigh-Bénard

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Rapidly rotating regime (Gastine et al., 2016):

$$Nu \sim Ra^{3/2} E^2 \tag{2}$$

“Ultimate regime”

Kraichnan (1962) : Deviation from classic 1/3 law, turbulent boundary layers

$$Nu \sim Ra^{1/2} (\ln Ra)^{-3/2} \tag{3}$$